P E T E R   S T R O H M A Y E R

Thoughts on relativistic acceleration

A uniformly accelerated point of matter reaches in infinitesimal steps from a slower inertial system to an event location of the next faster inertial system determined by the Lorentz transformation (L-T). It jumps, so to speak, from a synchronized clock of the preceding system to the passing, temporally more advanced encounter clock of the faster target system. The instantaneous jump of the matter point into the respective target system is followed by an (infinitesimal) period of time in which it rests from the point of view of the target system. The ratio of the difference in velocity to the duration of this period is the measure of acceleration. During this infinitesimal period of dwell, the subsequent encounter clock of the next faster target system has moved away from the common origin cover of all target systems, so that the time of the encounter clock of the next target system is already more advanced compared to the clock of the previous target system. This is followed by the next increase in speed and the subsequent time period, and so on.

During its jumps, the matter point retains its own time (the sum of the infinitesimal periods of time from the point of view of the target system reached in each case, in which it dwells for an infinitesimal period of time in each case) unaltered; it takes its own clock with it, so to speak, whose rate is not influenced by the acceleration. The clocks of the respective target systems are advanced to an ever greater extent than his own as the duration of the acceleration increases. This is also true in comparison to the clocks of the initial system, into which the event of the arrival of the accelerated is transformed back from the point of view of the target system reached last in each case by means of Lorentz transformation. The intermediate events at the end of the respective infinitesimal time periods in the reached velocity level of the target system are transformed back with Lorentz transformations into the initial system A, where - in the context of an iteration of all acceleration jumps - they together make up the world line of the acceleration from the point of view of A. The accelerated person's arrival at the end of the respective infinitesimal time periods in the reached velocity level of the target system is transformed back into the initial system A by means of Lorentz transformations. Thus, the accelerated is increasingly younger compared to an observer of the initial system.

Equal acceleration steps (equal increment steps of rapidity, see the essay Shadow Time in Shadow Space, SiS) and equal acceleration time spans from the point of view of the accelerated matter point mean uniform acceleration. The constant infinitesimal "perpendicular" world line sections of the acceleration phases from the point of view of the individual target systems (the passing of time without changing the location from the point of view of the target system) form the transmission belt for a calculation of the world line of acceleration and the "time saving" at the accelerated.

With the choice of the same velocity levels and the same acceleration phase, the acceleration a is fixed (e.g. 0.1 c/1sec). At each velocity level, the event of the end of the always identical time period z from the viewpoint of the target system (e.g. x=0, t=z=1) is to be reproduced in the initial system on the basis of the total velocity achieved (to be transformed with L-T). The extent of the individual path sections xA and the individual time sections tA of these transformations from each velocity step from the point of view of the initial system A are to be added there in each case to the coordinate value reached so far and result in the coordinates of the new event (of the location of the accelerated point of matter). In each velocity step, the same time span z (e.g. "1") elapses from the point of view of the accelerated person. The total proper time of the accelerated tB is the sum of these time spans z. The time tA from the point of view of the initial system A is advanced according to the sum of all transformations of these time spans.

Related to the always same segment on the perpendicular (temporal) component of a world line from the point of view of the respective target system with the temporal length "z", the L-T simplifies. From the point of view of the initial system, tAS is the time segment and xAS the path segment (both dependent on v of the respective target system), which is added in each case during an acceleration phase tB=z up to the next event point:

tAS=z/sqr(1-v2)

xAS=v*z/sqr(1-v2)

<u>Iteration of time segments</u>:

n = number of the acceleration step just completed

x = total number of acceleration steps completed

tB = z*n (time of the accelerated; proper time)

v = tanh(a*tB) = tanh(a*z*n)

tAS = z/sqr(1-(tanh(a*z*n))^2)

The total elapsed time from the point of view of A **tA** = sum tAS = z/sqr(1-(tanh(a*z*1))^2) + z/sqr(1-(tanh(a*z*2))^2) + ... + z/sqr(1-(tanh(a*z*n))^2).

$$tA=\sum_{n=1}^{x} \frac{z}{\sqrt{(1\text{-}(\tanh(a\cdot z\cdot n))^2)}} \;;\; a=0{,}1;\; z=1$$

<u>Iteration of the path segments</u>:

xAS = z*tanh(a*z*n)/sqr(1-(tanh(a*z*n))^2)

The total distance travelled from the point of view of A **xA** = sum xAS = z*tanh(a*z*1)/sqr(1-(tanh(a*z*1))^2) + z*tanh(a*z*2)/sqr(1-(tanh(a*z*2))^2) + ... + z*tanh(a*z*n)/sqr(1-(tanh(a*z*n))^2).

$$xA=\sum_{n=1}^{x} \frac{z\cdot\tanh(a\cdot z\cdot n)}{\sqrt{\left(1-(\tanh(a\cdot z\cdot n))^2\right)}} \;;\; a=0{,}1;\; z=1$$

It can be seen that a path segment xAs from the point of view of the system A is the time segment from the point of view Z (tBs) times the respective intrinsic velocity V (= v*Gamma) reached at a certain intrinsic time tB.

Both the coordinate velocity v and the intrinsic velocity V are equal from the reciprocal point of view of both systems. The intrinsic velocity V of a matter point is the time elapsed in the rest system of the accelerated Z ("own time") divided by the distance covered from the point of view of the initial system A ("foreign distance"), V=v/sqr(1-v2) (see SiS).

V = tanh(a*tB)/sqr(1-(tanh(a*tB))^2)

respectively

$$V=\frac{\tanh(a\cdot tB)}{\sqrt{\left(1-(\tanh(a\cdot tB))^2\right)}};\; a=0{,}1$$

The intrinsic velocity V forms the bridge from the time tB of the accelerated to the path xA and subsequently to the time tA from the point of view of the initial system.

<u>Determine the hyperbolic equation of motion</u>:

The calculation with the intrinsic velocity V makes it possible to determine the distance travelled from the point of view of system A directly from the past time from the point of view of the accelerated person. Starting from the respectively achieved intrinsic velocity V=f(a,tB) of the target system Z, the total distance xA is determined from the point of view of the initial system A at a certain intrinsic time of the accelerated tB.

The **path xA** from the point of view of the initial system as a **function of the time of the accelerated tB** is the integral over the respective infinitesimal time segments dtB from the point of view of the accelerated times the intrinsic velocities (which corresponds to the iteration of xA above):

$$xA=\int_0^{tB} dtB\left(\frac{\tanh(a\cdot tB)}{\sqrt{(1-(\tanh(a\cdot tB))^2)}}\right);\ a=0{,}1$$

or (https://www.integralrechner.de)

$$xA=\frac{\cosh(a\cdot tB)-1}{a};\ a=0{,}1$$

xA=(cosh(a*tB)-1)/a

The time tA from the point of view of the initial system as a function of the time of the accelerated tB is the integration of the individual infinitesimal time segments tB multiplied by the Lorenz factor corresponding to the velocity achieved (which corresponds to the iteration of tA above):

$$tA=\int_{0}^{tB} dtB\left(\frac{1}{\sqrt{(1\text{-}(\tanh(a\cdot tB))^{2})}}\right);\ a=0{,}1$$

or (https://www.integralrechner.de)

$$tA=\frac{\sinh(a*tB)}{a};\ a=0{,}1$$

tA=sinh(a*tB)/a

With a "speed of light" of 299 792 458 m/s, the average acceleration due to gravity g of 9.81 m/s2 corresponds to an acceleration of 0.00000003272 c/s. If I were to accelerate to this extent in gravity-free space, I would remain about one tenth of a second younger per day (tB=86400 seconds) than an unaccelerated observer according to this formula.

Reversing this formula, I now determine the time of the accelerated tB as a function of the time of the initial system tA (http://rechen-fuchs.de/gleichungen-loesen-online/).

$$tB=\frac{(\text{asinh}(a\cdot tA))}{a};\ a=0{,}1$$

tB=(asinh(a*tA))/a

I substitute the time of the accelerated tB into the velocity function v=tanh(a*tB)and get the velocity v from the point of view of the initial system as a function of the time from the point of view of the initial system tA, where a is truncated:

$$v=\tanh(\text{asinh}(a*tA));\ a=0{,}1$$

or (https://www.integralrechner.de)

$$v=\frac{a\cdot tA}{\left(\sqrt{(a^2\cdot tA^2+1)}\right)};\ a=0{,}1$$

Integration over the time segments dtA times v yields the path from the point of view of the initial system xA as a function of the time from the point of view of the initial system tA:

$$xA=\int_0^{tA} dtA\left(\frac{a\cdot tA}{\sqrt{(a^2\cdot tA^2+1)}}\right)\ ;\ a=0.1$$

or (https://www.integralrechner.de)

$$xA=\frac{\sqrt{(a^2\cdot tA^2+1)}-1}{a};\ a=0,1$$

xA=((sqr(a2*tA2+1))-1)/a

This is the sought hyperbolic world line from the point of view of the initial system at a uniform acceleration from the point of view of the accelerated.

Excursus concerning the relation to the spatiotemporal interval:

While investigating Bell's acceleration ("spaceship paradox") I accidentally came across the following connection: A spaceship S accelerated with a=0.1 at a distance of 10 from the origin has the coordinates 10/0 from the point of view of each of the target systems in question at each initial time tA (an acceleration of a=0.2 would result in the same constellation at an initial distance of 5).

The coincidence consisted in the fact that I had chosen the distance of the spaceships with 10 and thus unconsciously the reciprocal of the chosen acceleration a of 0.1. From the point of view of all target systems, this yielded the result that the front (right) 10 distant spaceship, which from the point of view of the respective system had already started acceleration earlier, is at the time tZ=0 (at origin cover) always in the same original rest distance of 10 from the origin of this target system.

Thus, if the spatial distance of the spacecraft from the origin of the source system is as large as the reciprocal of the self-acceleration, then from the point of view of each of the Zs in question, the world line begins at tZ=0 (at the spatial distance corresponding to the acceleration). From the point of view of each target system, the coordinates of the accelerated matter point change virtually nothing and appear to remain at the initial value of 10/0. In a sense, the matter point jumps from a location 10/0 in one Z to the "same" location 10/0 in the next Z. This is not to be interpreted as meaning that it would remain "there" statically. This is shown by the back transformations of the respective locations 10/0 (from the point of view of the respective target systems) into the initial system. Conversely, the "world point" of the defined event from the point of view of the source system A 10/0 does not coincide with the "world points" of the events from the point of view of Z 10/0 (i.e. when all A and Z are in origin cover). Rather, the transformations of 10/0 from the point of view of the initial system into the individual superimposed target systems Z lie on a hyperbola that resembles the hyperbolic world line of an accelerated motion.

According to the pseudo-Riemannian metric, expressed in terms of the hyperbola x2-t2=s2 , a (spatiotemporal) spatiotemporal interval of s=10 leads to a function,

$$x=\sqrt{s^2+tA^2};\ s=10$$

which coincides with the acceleration formula derived above for the 10 distant spaceship S. The same correspondence of the acceleration hyperbola with the metric results for a pure spatial distance s=5 for an acceleration 0.2 and so on.

This connection between an acceleration and a spatiotemporal distance becomes somewhat clearer with a view to a spatiotemporal distance between two events. Each space-like spatiotemporal distance can be assigned the reference velocity vR of a sought system, from whose point of view the distance of the chosen event E from the origin becomes a purely spatial one. vR results from the ratio of t/s (see SiS, p 40).

One can reinterpret this geometrical connection existing with uniform movements and the pure spatial distance resulting from it into an accelerated movement of a point of matter starting from the origin in this spatial distance, at the beginning of which the time t and thus also the reference velocity vR is 0. If in the course of the time t the velocity is increased in the direction of motion to the left (-v) under constant consideration of the ratio t/s and under constancy of the spatiotemporal distance defined with the choice of the pure spatial distance (according to the rules of the Lorentz transformation), we obtain a target system Z moving with the respectively achieved velocity -v, from whose point of view the event E with the coordinates t and s can be interpreted as the achieved position of the motion of a point of matter.

The position of the initial event transformed into this target system (with its pure spatial distance given from the point of view of the initial system A at time 0) is interpreted as the world point of a matter point which, from the point of view of the target systems Z - superimposed with origin cover - has moved in the opposite direction to the right with velocity +v to this world point in the time span t shown in them. From the point of view of Z, this fictitious movement of the matter point begins at the original pure spatial distance (from the point of view of A) at time t=0 and describes an acceleration hyperbola as a world line. The fiction of a movement from the point of view of all superimposed Z originates from the fact **that here not the matter points but the coordinate systems Z1 to Zn in the opposite direction are considered to be accelerated (with the velocities -v achieved)** and from their point of view it is judged where the matter point at rest at distance 10 from point of view A is located from the point of view of these changing Z, which altogether form an "**accelerated coordinate system**". The locations of the initial event E from the point of view of all Z are mentally superimposed and summarized in a system, from which the world line of an accelerated motion of a point of matter results.

A continuous increase of t is accompanied by a discontinuous increase of -v. If one refers to the intrinsic velocity V, which is larger by the factor gamma (Lorentz factor), then t and -V increase proportionally, which corresponds to a uniform acceleration of Z. The change-segments of the way xAS and the change-segments of the time tAS in dependence on tA correspond to those, which were also determined by the iteration and later the integration at motion of an accelerated matter-point with +v from the point of view of a resting initial system.

Thus the accelerated system Z to the resting matter point behave like an accelerated matter point to the resting initial system. This with the difference that the effect of acceleration from the point of view Z depends on an original pure spatial distance and corresponds to the inverse of this distance. If the distance goes towards zero, the acceleration becomes greater and greater. However, its course also becomes mathematically more and more extreme. At a distance of zero, even if the relative velocity of the Z is as high as possible, there is no longer any temporal component. The matter point has an infinitely high acceleration, but no more time passes where this could manifest itself in a reached spatial distance. At initial distances just above the spatial distance zero, the acceleration process consumes almost no time. Only just before v=c the time (and also the space) speed up towards infinity.

The greater the initially selected pure spatial distance, the less the distance s travelled on the x-axis is added in relation to the time t (from the point of view of Z) increasing from zero, which is to be understood as a correspondingly slower acceleration of Z . This leads to the connection that when using a spatiotemporal interval as a representation of an accelerated movement, the measure of acceleration can be determined with the reciprocal value of the pure spatial distance.

In order to describe with the function derived from the metric an accelerated motion not at a spatial distance, but (with a function graph) an accelerated motion starting from the origin, the originally chosen spatial distance from the origin, i.e. the inverse acceleration 1/a, has to be subtracted from the value x:

xA=sqr(1/a2+tA2)-1/a

$$xA=\sqrt{s^2+tA^2};\ s=10$$

This corresponds to the hyperbolic equation of motion derived above

xA=(sqr(a2*tA2+1)/a)-1/a

$$xA=\frac{\sqrt{\left(a^2\cdot tA^2+1\right)}-1}{a};\ a=0{,}1$$

The hyperbolic equation of motion based on the spatiotemporal interval is:

$$xA=\sqrt{\frac{1}{a^2}+tA^2};\ a=0{,}1$$

respectively

$$tA=\sqrt{\left(\frac{1}{a^2}-xA^2\right)};\ a=0,1$$

The expression 1/a2 is the constant corresponding to the spatiotemporal distance. For uniform acceleration, the ratio a=sqr(1/(xA2-tA2) applies.

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